UNNS and Carroll's Paradox: Recursive Justification and Repair Stabilization

UNNS Research Notes

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Abstract

Carroll's paradox ("What the Tortoise Said to Achilles", 1895) highlights the infinite regress of justification in logic: every logical rule, once accepted, seems to demand acceptance of a higher-order rule incorporating it. Through the lens of the Unbounded Nested Number Sequence (UNNS) framework, this paradox is natural: logic behaves recursively. Crucially, UNNS introduces stability operators—particularly repair and normalization—that tame the regress by collapsing it into a fixed point (the "zero nest"). We formalize this view and provide a visualization of justification spiraling inward to stability.

1 Carroll's paradox

In Carroll's dialogue, Achilles accepts premises A, B and the rule

$$\text{If } (A \wedge B) \Rightarrow Z.$$

The Tortoise insists Achilles must also accept the higher-order premise:

$$(A \wedge B) \wedge ((A \wedge B) \Rightarrow Z) \Rightarrow Z,$$

and so on. This creates an infinite regress: each justification demands another.

2 UNNS interpretation

UNNS interprets this regress as a *recurrence*:

$$J_{n+1} = f(J_n),$$

where J_n is the n-th justification. The paradox is that J_n never terminates. But in UNNS, recursion is the natural substrate. Infinite regress is expected.

The problem arises only if no stabilizing operator is applied. UNNS introduces repair rules: they collapse runaway recursion back into the substrate vacuum, $\mathbf{0}$, or a fixed-point justification.

3 Stabilization lemma

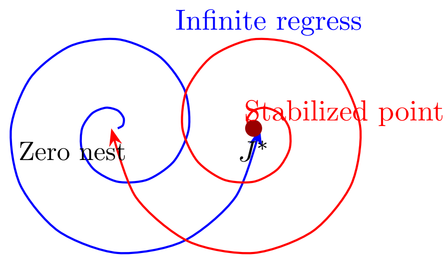
Lemma 3.1 (Justificatory recursion stabilizes under UNNS repair)**.** *Let (J_n) be the justificatory sequence defined recursively by $J_{n+1} = f(J_n)$. Suppose there exists a repair operator $\mathcal{R}$ such that $\mathcal{R}(J_n) = J_k$ for some $k < n$ whenever J_n exceeds a stability threshold (e.g. recursion depth or residue norm). Then (J_n) stabilizes to a fixed point J^* after finitely many applications of $\mathcal{R}$.*

Proof. By assumption, $\mathcal{R}$ maps runaway justifications back to an earlier element. Thus the sequence becomes eventually periodic. Repair contracts the cycle into a fixed point $J^* = \mathcal{R}(J^*)$. Hence recursion stabilizes, preventing infinite regress. □

Remark 3.2. *In logical terms, UNNS repair means: one does not demand endless acceptance of higher-order rules; instead, once the rule has been embedded, recursion collapses into a stable justification state.*

4 Visualization

The paradox may be visualized as a spiral of justifications. Without repair, it spirals outward indefinitely. With repair, the spiral is re-routed inward, collapsing into a fixed point: the "zero nest" of justification.



5 Conclusion

Carroll's paradox exposes the recursive nature of logic. UNNS reframes the regress not as a contradiction, but as a recursive nest that can be stabilized through repair. Infinite regress thus becomes a constructive feature of the logical substrate, revealing recursion at work.